THE FINNISH COORDINATE REFERENCE SYSTEMS

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1. Introduction

The realization of a geoinformation system requires that the geodetic reference system is defined and the measurements are carried out in the chosen system. The IAG Subcommission for Europe (EUREF) has worked vigorously to further the adoption of the ETRS89 datum all over Europe, and this work has been carried out in cooperation with the National Mapping Agencies (NMAs). The Spatial Reference Workshop organized by MEGRIN in 1999 also recommended that ETRS89 be implemented in all member states of the European Union (EU). In addition to this, the recommendations of the Workshop called for the parameters and algorithms for transforming from ETRS89 to the national coordinate frame, providing coordinates with an accuracy 1 to 2 m, to be placed in the public domain (Ihde *et al.*, 2000).

In Finland the measurements required for the realization of ETRS89 started as early as in 1989. Since then several GPS densifications have been made in order to achieve improved accuracy. The EUREF Subcommission accepted the GPS measurements performed in 1996 - 1997 as an official extension of EUREF. The campaign carried out is referred to as EUREF-FIN.

The Finnish Geodetic Institute (FGI) and the National Land Survey of Finland (NLS) are responsible for creating and maintaining the national reference frames used for national mapping purposes. Replacing a national coordinate frame with a new one is a major undertaking that takes decades to complete. In Finland the work started in 1998, when FGI set up a working group with the task to establish whether the geodetic reference frame and the height system used at the time needed to be renewed, and to prepare a proposal based on its findings. The working group consisted of specialists from a wide range of institutions within the field of surveying and mapping. In its proposal the working group recommended the introduction of a new geodetic reference frame based on ETRS89 along with a new height system relying on the results of the third precise levelling (Anon. ,1999).

The Finnish Ministry of Agriculture and Forestry incorporated the recommendations made by the working group into its strategy for public surveying for 2001 - 2010. FGI and NLS then assigned a special working group to prepare more specific recommendations and proposals on how to carry out the transformation from the reference frame used at the time to the new one. The working group accordingly published two recommendations titled JHS153 and JHS154.

This paper deals with the main items of the above-mentioned two recommendations. In addition, the national reference frames previously in use and the creation of the EUREF-FIN frame are briefly described, including the methods and parameters for performing the transformations between the new and the old frame.

2. Reference ellipsoids and map projections used in Finland

2.1 Introduction

On maps the true shape of the Earth, mathematically described as a reference ellipsoid, is commonly represented as a plane on which every point of the Earth's surface is projected. The projection can be accomplished in a number of ways. The Earth's surface can be directly projected on to a plane (azimuthal projection) or first on to a cone or cylinder, which is then opened to make a plane (conic or cylindrical projection). Common to all of these projections is that the true shape of the Earth can only be approximately represented.

2.2 The Hayford ellipsoid and Transverse Mercator projection (Gauss-Krüger)

When the Finnish Geodetic Institute (FGI) started carrying out triangulation in 1919 in Finland, the Hayford ellipsoid (dimensions determined by John F. Hayford in 1909) was chosen as the reference ellipsoid. The dimensions of the ellipsoid, also known as the International ellipsoid 1924, are as follows:

- a = 6378388.0 m
- 1/f = 297.0

In 1922 the Transverse Mercator projection with the Gauss-Krüger grid (also referred to as the Gauss-Krüger projection in this text) was selected for mapping purposes. The Gauss-Krüger projection is a transverse cylindrical projection, the axis of the cylinder being at right angles to the axis of the Earth. The projection is characterized by the following:

- The axis of the cylinder lies in the plane of the equator and forms the y-axis (easting) of the horizontal coordinate reference system.
- The cylinder touches the reference ellipsoid along a great circle (the central meridian). On the plane the central meridian is a straight line and its length is true (scale = 1.0). The central meridian is the x-axis of the horizontal coordinate reference system.
- The other meridians are not straight but slightly curved lines that intersect the central meridian at the pole.
- The parallels represented as slightly curved lines are orthogonal to the meridians.
- The scale, shape, area and bearing are all projected with very little, if any, distortion.
- The scale factor due to the round shape of the Earth increases related to the distance from the central meridian. The effect of this error is reduced by use of narrow projection zones (3° wide in the Gauss-Krüger projection).

The Hayford ellipsoid and the Transverse Mercator projection with the Gauss-Krüger grid are associated with Finnish coordinate reference systems like vvj and kkj (described in more detail later in this text) and with the coordinate reference systems of some municipalities.

2.3 The GRS80 ellipsoid and UTM projection

In 1979 the International Union of Geodesy and Geophysics (IUGG) introduced the Geodetic Reference System 1980 (GRS80), which was based on the theory of the geocentric equipotential ellipsoid (Moritz, 2000). The defining constants of GRS80 are as follows:

- $a = 6378137.0$ m — equatorial radius of the Earth
- $GM = 398600.5 \times 10^9$ m^3s^{-2} — geocentric gravitational constant of the Earth
- $J_2 = 1082.63 \times 10^{-6}$ — dynamical form factor of the Earth
- $\omega = 7.292115 \times 10^{-5}$ rad s^{-1} — angular velocity of the Earth

All geometric parameters describing the reference ellipsoid can be derived from the above parameters, among these the reciprocal flattening, which has the following value:

- $1/f = 298.257222101$ — reciprocal flattening

The Finnish coordinate frame EUREF-FIN is a realization of the European ETRS89 coordinate reference system, which is associated with the GRS80 ellipsoid. The geometric parameters of the reference ellipsoid used in EUREF-FIN are thus the same as in ETRS89, i.e. the parameters of GRS80.

For national mapping purposes the Universal Transverse Mercator (UTM) projection or grid system was adopted in 2003. Although the UTM projection is a transverse cylindrical projection like the Gauss-Krüger projection used in Finland, there are some differences between the two. The UTM projection is characterized by the following:

- The axis of the cylinder is in the plane of the equator and forms the y-axis (easting) of the horizontal coordinate reference system.
- The cylinder cuts the reference ellipsoid along two meridians. The scale factor at the central meridian is 0.9996.
- The UTM projection is defined for areas between latitudes 80°S and 84°N.
- The projection (grid) zones are 6° wide, and numbered 1 through 60 (starting from 180°W and proceeding eastwards to 180°E). For example, zone 35 is 24°E to 30°E with a central meridian of 27°E.
- Coordinate values at the origin of each zone are 0 m for the northing and 500,000 m for the easting in the Northern Hemisphere.
- Several reference ellipsoids can be used with the UTM projection.

It should, however, be noted that the UTM projection is used with some modifications in Finland. To fit the whole country in a single zone a wider projection zone is used instead of the standard 6° zone. When extensive areas are concerned, the normal false easting value of 500,000 m can be replaced by 8,500,000 m in order to avoid negative coordinates.

3. The ETRS89 coordinates in Finland

ETRS89 (European Terrestrial Reference System) is a three-dimensional coordinate reference system that is fixed to the stable part of the Eurasian plate and is coincident with ITRS (International Terrestrial Reference System) at epoch 1989.0. The realization of ETRS89 requires GPS measurements to be carried out across Europe. For this purpose the first EUREF GPS campaign was initiated in 1989, resulting in a rather sparse network of points. However, a number of European countries subsequently densified the network within their own borders.

In the following chapters the steps involved in creating the EUREF-FIN frame will be described in brief.

3.1 The EUREF89 campaign of 1989

The EUREF GPS campaign carried out in 1989 involved four stations in Finland. The overall objective of the campaign was to establish a common European coordinate reference system, although at the time the campaign was limited to the countries of Western Europe. The number of stations measured all over Europe totalled 93. The campaign was the first realization of the ETRS89 coordinate reference system. ETRS89 coincides with the ITRF89 system at the epoch 1989.0 (Gurtner *et al.,* 1992).

Some details of the campaign as carried out in Finland:

- Number of stations: 4
- Observation time: 6 x 5 hours (May 16 - 28, 1989)
- Processing software: Bernese software
- Accuracy of the stations: ±1 cm (latitude, longitude), ±2 cm (height)

3.2 The GPS-Suomi campaign of 1992

Following the EUREF89 campaign it was decided that a densification of the network was to be carried out jointly by the Finnish Geodetic Institute (FGI), the National Board of Survey (NBN) or National Land Survey of Finland (NLS), as it is currently named, and the National Board of Navigation (NBN), later renamed the Finnish Maritime Administration (FMA). The coordinates that had been obtained from three stations in connection with the EUREF89 campaign were held fixed in the adjustments (Ollikainen, 1993a, 1993b).

Some details of the campaign:

- Number of stations: 22
- Observation time: at least 2 days (2 x 11.5 hours) per station
- Processing software: GPPS, FILLNET, Bernese
- Accuracy (RMS) of the coordinates: ±13 mm, ±18 mm, ±42 mm in North, East and Up component

3.3 The permanent Finnish GPS network – FinnRef ®

In 1993 the Finnish Geodetic Institute started to plan and establish a permanent GPS network. The construction of 12 stations was completed in 1996, and the resulting network was called FinnRef. The purpose of the network is to serve as a base for the national coordinate reference system and to link local reference frames to international ones. The network also has scientific applications; it is, for instance, used for monitoring the Fenno-Scandian land uplift (Ollikainen *et al.,* 1997).

FinnRef is part of the Nordic GPS network. Four stations also belong to the European Permanent Network (EPN) and one to the International GPS Service (IGS). All permanent stations are official EUREF points accepted by the IAG Subcommission for Europe.

Some details of the network:

- Number of stations: 12
- Observation time: continuously operating
- Processing software: Bernese software
- Accuracy (RMS) of the coordinates: ±1 mm, ±1 mm, ±2 mm North, East and Up component

3.4 The EUREF-FIN campaign of 1996 - 1997

When the construction of the FinnRef stations had been completed, and the network was fully operating, it was decided that the EUREF points in Finland were to be further densified. The densification was carried out by FGI in 1996 - 1997. Most of the stations observed were previous first-order triangulation points, but a few tide gauge sites and precise levelling benchmarks were also included. The permanent FinnRef network served as a backbone for the densification and as a tight link to the European (ETRS) and international (ITRS) reference frames. The Finnish coordinate frame EUREF-FIN is based on the observations made in connection with this campaign (Ollikainen *et al.* 2000).

Some details of the campaign:

- Number of stations: 100
- Observation time: 2 x 24 hours
- Processing software: Bernese software
- Accuracy (RMS) of the coordinates: ±2 mm, ±2 mm and ± 6 mm in North, East and Up component

3.5 The densification of the EUREF-FIN network in 1998 - 1999

In 1998 a new densification campaign was initiated, as the EUREF points measured in 1996 - 1997 and totalling 100 were not sufficient in practical terms. It was consequently decided that a network of 350 points was to be created since these would be easy to use (thanks to roads in the vicinity). The stations set up in connection with the previous densifications consisted primarily of triangulation points located on hilltops. In this campaign the points were determined jointly by FGI, NLS, FMA and some other organizations. The measurements were, however, mainly carried out by FGI (Ollikainen *et al.,* 2001).

Some details of the campaign:

- Number of stations: 350
- Observation time: 6 hours
- Processing software: Pinnacle, Global X (adjustment)
- Accuracy (RMS) of the coordinates: ±4 mm, ±4 mm and ± 6 mm in North, East and Up component

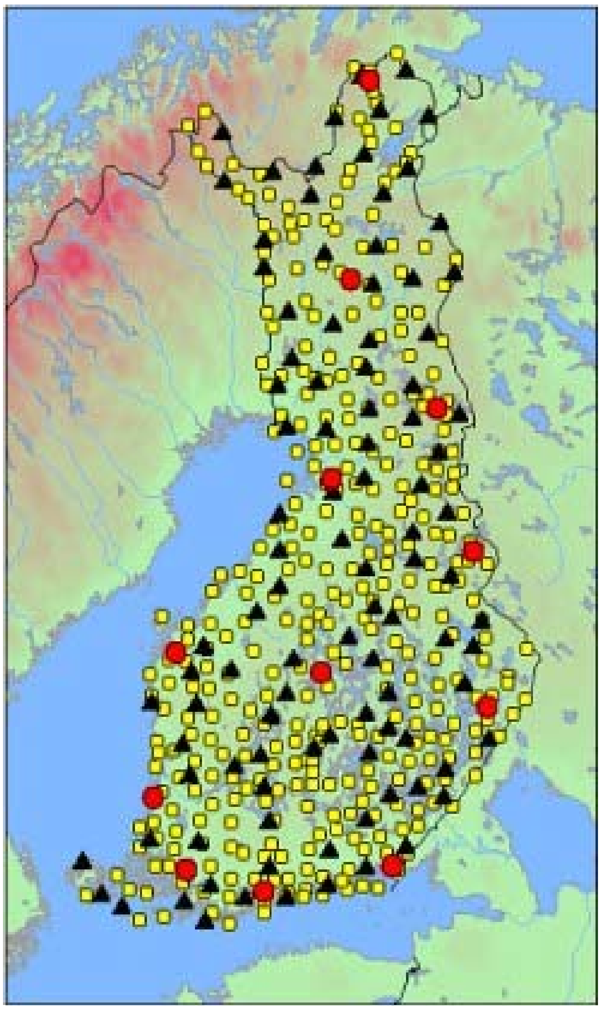

Figure 1. EUREF-FIN points measured by FGI. The red dots on the map indicate the permanent FinnRef network. The points established in connection with the first EUREF-FIN densification (1996-1997) are marked with black triangles, whereas points of the second EUREF-FIN densification (1998-1999) are marked with yellow squares.

3.6 Plans for the future

NLS and FMA (NBN) have already started densifying the network currently consisting of 450 points measured by FGI. There are plans to extend the EUREF-FIN network with several thousands of new points in order to facilitate the use and maintenance of the current Finnish coordinate frame.

4. New horizontal coordinate reference systems

In the resolutions adopted in 1998 the Nordic Geodetic Commission recommended that the ETRS89 coordinate reference system and the UTM projection be introduced in the Nordic countries. Furthermore, the UTM-type ETRS-TMn projection has also been recommended within EU for large-scale mapping. In 1998 a national working group was set up with the task to establish the need for a new coordinate frame in Finland. The task also involved determining the benefits and drawbacks of such a frame, the course of action and the timetable for its introduction. The working group issued its report in 1999. It was then decided that the work was to be continued in a smaller working group consisting of experts from FGI and NLS. The results of the work performed by this group were published in the recommendations for the public administration JHS153 and JHS154 (Anon. 2002, 2003).

JHS153 recommends that the ETRS89 coordinate reference system be used within the public administration in Finland. It also defines the three-dimensional transformation between the EUREF-FIN and kkj frames.

The recommendation JHS154 again determines the horizontal coordinate frames for Finland based on EUREF-FIN coordinates and UTM or Gauss-Krüger projection.

4.1 The ETRS-TM35FIN projection

The ETRS-TM35FIN coordinates are based on the UTM projection and the GRS80 ellipsoid. As a deviation from the standard UTM, a wider projection zone is, however, used instead of the standard 6° wide zone.

The N-coordinate (northing) is the distance from the equator, and the E-coordinate (easting) is the distance from the central meridian. The easting of the central meridian is set to 500,000 m in order to avoid negative values. The value of 8,500,000 m can also be used for the false easting in special cases where the eastings would otherwise become negative. The scale at the central meridian is 0.9996.

The parameters of the ETRS-TM35FIN projection are presented below according to the ISO 19111 standard (spatial referencing by coordinates).

ETRS-TM35FIN Description	
Entity	**Value**
CRS ID	ETRS-TM35FIN
CRS alias	ETRS89 Transverse Mercator CRS for Finland
CRS valid area	Finland
CRS scope	CRS for mapping in Finland, national level
Datum ID	ETRS89, EUREF-FIN
Datum alias	European Terrestrial Reference System 1989
Datum type	Geodetic
Datum realization epoch	1989.0
Datum valid area	Finland / EUREF

ETRS-TM35FIN Description	
Entity	**Value**
Datum scope	European datum consistent with ITRS at the epoch 1989.0 and fixed to the stable part of the Eurasian continental plate for georeferencing of GIS and geokinematic tasks
Datum remarks	See Boucher, C., Altamimi, Z. (1992): The EUREF Terrestrial Reference System and its First Realizations. Veröffentlichungen der Bayerischen Kommission für die Internationale Erdmessung, Heft 52, München 1992, pages 205-213- or - ftp://lareg.ensg.ign.fr/pub/euref/info/guidelines
Prime meridian ID	Greenwich
Prime meridian Greenwich longitude	0°
Ellipsoid ID	GRS 80
Ellipsoid alias	New International
Ellipsoid semi-major axis	6 378 137 m
Ellipsoid shape	True
Ellipsoid inverse flattening	298.257222101
Ellipsoid remarks	See Moritz, H. (1988): Geodetic Reference System 1980. Bulletin Geodesique, The Geodesists Handbook, 1988, Internat. Union of Geodesy and Geophysics
Coordinate system ID	TM35FIN
Coordinate system type	Projected
Coordinate system dimension	2
Coordinate system axis name	N
Coordinate system axis direction	North
Coordinate system axis unit identifier	Metre
Coordinate system axis name	E
Coordinate system axis direction	East
Coordinate system axis unit identifier	Metre
Operation ID	TM35FIN
Operation valid area	Finland
Operation scope	For conformal mapping in Finland, national level
Operation method name	Transverse Mercator Projection
Operation method formula	Transverse Mercator Mapping Equations, in Hooijberg, Practical Geodesy, 1997, p. 81-84, 111-114.
Operation method parameters number	7
Operation parameter name	Latitude of origin
Operation parameter value	0° , the Equator
Operation parameter name	Longitude of origin
Operation parameter value	Central meridian 27°E
Operation parameter remarks	Central meridian (CM) of the zone
Operation parameter name	False northing
Operation parameter value	0 m
Operation parameter remarks	
Operation parameter name	False easting
Operation parameter value	500 000.0 m
Operation parameter remarks	
Operation parameter name	Scale factor at central meridian
Operation parameter value	0.9996

ETRS-TM35FIN Description	
Entity	**Value**
Operation parameter name	Width of zone
Operation parameter value	The Finnish Territory
Operation parameter name	Latitude limits of system
Operation parameter value	84°N and 0°N
Operation parameter remarks	

4.2 The ETRS-GKn projection

In addition to ETRS-TM35FIN, another projection method is used in Finland, although mainly within municipalities and for local undertakings like for instance construction work and town planning. The ETRS-GKn, where n means the central meridian (27 for 27°E etc.), is a Transverse Mercator projection that is associated with the Gauss-Krüger grid based on EUREF-FIN and the GRS80 ellipsoid .

As in ETRS-TM35FIN, the N-coordinate (northing) of the ETRS-GKn is the distance from the equator and the E-coordinate (easting) is the distance from the central meridian. The false easting is also set to 500,000 m. The scale at the central meridian is 1.0.

The parameters of the ETRS-GKn projection are presented below according to the ISO 19111 standard (spatial referencing by coordinates).

ETRS-GKn Description	
Entity	**Value**
CRS ID	ETRS-GKn
CRS remarks	n is the used central meridian in degrees (19-31)
CRS alias	ETRS89 Gauss-Krüger CRS for Finland
CRS valid area	Finland
CRS scope	CRS for large scale mapping in Finland
Datum ID	ETRS89, EUREF-FIN
Datum alias	European Terrestrial Reference System 1989
Datum type	Geodetic
Datum realization epoch	1989.0
Datum valid area	Finland / EUREF
Datum scope	European datum consistent with ITRS at the epoch 1989.0 and fixed to the stable part of the Eurasian continental plate for georeferencing of GIS and geokinematic tasks
Datum remarks	See Boucher, C., Altamimi, Z. (1992): The EUREF Terrestrial Reference System and its First Realizations. Veröffentlichungen der Bayerischen Kommission für die Internationale Erdmessung, Heft 52, München 1992, pages 205-213- or - ftp://lareg.ensg.ign.fr/pub/euref/info/guidelines
Prime meridian ID	Greenwich
Prime meridian Greenwich longitude	0°
Ellipsoid ID	GRS 80
Ellipsoid alias	New International
Ellipsoid semi-major axis	6 378 137 m
Ellipsoid shape	True

ETRS-GKn Description	
Entity	**Value**
Ellipsoid inverse flattening	298.257222101
Ellipsoid remarks	See Moritz, H. (1988): Geodetic Reference System 1980. Bulletin Geodesique, The Geodesists Handbook, 1988, Internat. Union of Geodesy and Geophysics
Coordinate system ID	GKn
Coordinate system type	Projected
Coordinate system dimension	2
Coordinate system axis name	N
Coordinate system axis direction	North
Coordinate system axis unit identifier	Metre
Coordinate system axis name	E
Coordinate system axis direction	East
Coordinate system axis unit identifier	Metre
Operation ID	GKn
Operation valid area	Finland
Operation scope	For conformal mapping in Finland, sub-national level
Operation method name	Gauss-Krüger Projection
Operation method formula	Transverse Mercator Mapping Equations, in Hooijberg, Practical Geodesy, 1997, p. 81-84, 111-114.
Operation method parameters number	7
Operation parameter name	Latitude of origin
Operation parameter value	0° , the Equator
Operation parameter name	Longitude of origin
Operation parameter value	Central meridian 19°E, 20°E, 21°E…31°E
Operation parameter remarks	Central meridian (CM) of the zone
Operation parameter name	False northing
Operation parameter value	0 m
Operation parameter remarks	
Operation parameter name	False easting
Operation parameter value	500 000.0 m
Operation parameter remarks	
Operation parameter name	Scale factor at central meridian
Operation parameter value	1.00
Operation parameter name	Width of zone
Operation parameter value	1°
Operation parameter name	Latitude limits of system
Operation parameter value	84°N and 0°N
Operation parameter remarks	

5. Previous horizontal coordinate reference systems

Several horizontal coordinate reference systems have previously coexisted in Finland.

For nationwide use, there were originally two systems, namely vvj and kkj. Although still in use in many municipalities, the vvj grid coordinate reference system, which is the older one of the two, has since been replaced by the kkj grid coordinate reference system on the national level. In addition to these, some municipalities have introduced coordinate frames of their own (these are, however, not dealt with in this text).

5.1 The vvj grid coordinate system - *vanha valtion järjestelmä*

In the 1920's a reference ellipsoid, map projection and coordinate grid had to be chosen for national mapping purposes. As mentioned above, it was consequently decided that the Hayford ellipsoid and the Gauss-Krüger projection were to be used.

The Finnish Geodetic Institute (FGI) had been established in 1918 to head and perform the geodetic, astronomical and gravimetrical work in Finland, and to thus deliver the data required for producing nationwide maps. FGI accordingly started carrying out first-order triangulation in 1920. As the need for new maps was pressing, it was decided that giant loops were to be measured instead of an entire triangulation network. The loops were later filled with lower-order triangulation networks, the work of which was carried out by NLS.

As the initial coordinates for the triangulation, the astronomical coordinates of the observatory of Helsinki were used, and for this reason the vvj frame is also known as the Helsinki system. The azimuth between two triangulation stations (Kallio Church and Falkberg) was defined as the initial azimuth of the triangulation. However, since the deflections of the vertical were not known at the time, the vvj coordinates obtained differ from the Earth-centred coordinates by approximately 100 m.

New coordinates were needed urgently, and the loops were therefore adjusted as soon as the measurements had been carried out. When a new loop had been completed, it was adjusted together with the loops that had already been adjusted. In the adjustment the coordinates of the points in the loops already adjusted were held fixed. Thus the triangulation proceeded loop by loop. The coordinate reference system created in this fashion was not homogenous, nor free from errors, but it was nevertheless the first national coordinate reference system adopted in Finland.

5.2 The kkj grid coordinate system - *kartastokoordinaattijärjestelmä*

Until the time of writing, the horizontal coordinates in Finland have been based on the National Grid Coordinate System, referred to as *Kartastokoordinaatti-järjestelmä* (kkj) in Finnish. The multi-zone Basic Coordinate System (*Perus-koordinaatisto*) and the Uniform Coordinate System (*Yhtenäiskoordinaatisto*) composed of one 27^o wide zone have been used for topographic maps. In the near future ETRS-TM35FIN will, however, replace the old frames on printed maps (Anon., 1984).

In 1970 the kkj replaced the previous national grid system known as vvj. When the Finnish Geodetic Institute had carried out observations of the first-order triangulation network, the network was adjusted by use of one fixed point (the so-called common adjustment of 1966). The European Datum 1950 (ED50) coordinates of that particular point were held fixed. This meant that the coordinates of the first-order triangulation network could be introduced in the approximate ED50 system. The differences between the ED50 coordinates (transformed to the Gauss-Krüger projection plane) and the vvj coordinates were, however, so great that they were clearly visible on the topographic maps that had been printed. To avoid any further problems the horizontal ED50 coordinates were consequently transformed to better match the vvj frame. The transformation parameters for the horizontal coordinates of the ED50 and vvj frames were determined, and the coordinates thus transformed defined the new kkj frame.

In the Basic Coordinate Frame, Finland is divided into six projection zones, each 3° wide. The corresponding central meridians are 18°E (zone 0), 21°E (zone 1), 24°E (zone 2), 27°E (zone 3), 30°E (zone 4) and 33°E (zone 5). The grid of the basic coordinate frame is directed along the central meridian. The origin is the intersection of the equator and the central meridian. Thus the x-coordinate (northing) is the distance from the equator, and the y-coordinate (easting) is distance from the central meridian. To avoid negative values for the y-coordinates the easting of the central meridian is set to 500,000 m. Furthermore, the y-coordinates of the different zones are distinguished by the ordinal number of the particular zone inserted before the actual coordinate value. Hence the false eastings of the central meridians are 18°E = 500,000 m, 21°E = 1,500,000 m, 24°E = 2,500,000 m, 27°E = 3,500,000 m, 30°E = 4,500,000 m and 33°E = 5,500,000 m.

In the Uniform Coordinate Frame, Finland is contained in a single projection zone. The central meridian is 27°E. Regarding other details the Uniform Coordinate Frame correspond to the Basic Coordinate Frame, zone 3.

On topographic maps the Basic Coordinate Frame is in black, whereas the Uniform coordinate reference system is in red. In the third zone (27°E) the coordinate reference systems coincide, and the markings are in red.

6. Transformation between the EUREF-FIN and kkj coordinates

Three- and two-dimensional transformations between EUREF-FIN and kkj are determined in the recommendations JHS153 and JHS154. The transformation parameters for the 3D transformation are given in JHS153, and the parameters for the 2D transformation in JHS154.

6.1 The 3D transformation

In JHS153 the 7 parameters of the similarity transformation between EUREF-FIN and the kkj coordinates are presented. The transformation formula (Bursa-Wolf model) is as follows:

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix}_{kkj} = (1+m) \cdot \begin{vmatrix} 1 & \varepsilon_Z & -\varepsilon_Y \\ -\varepsilon_Z & 1 & \varepsilon_X \\ \varepsilon_Y & -\varepsilon_X & 1 \end{vmatrix} \cdot \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}_{EUREF-FIN} + \begin{vmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{vmatrix},$$

where the transformation parameters are:

		EUREF-FIN→kkj	kkj→EUREF-FIN
ΔX	Translation	96.0610 m	-96.0617 m
ΔY	Translation	82.4298 m	-82.4278 m
ΔZ	Translation	121.7485 m	-121.7535 m
ε_X	Rotation	4.80109 “	-4.80107 “
ε_Y	Rotation	0.34546 “	-0.34543 “
ε_Z	Rotation	-1.37645 “	1.37646 “
m	Scale difference	-1.49651 ppm	1.49640 ppm

In the formula the unit of the rotations is radian, whereas in the table the rotations are expressed in arc seconds. The accuracy of the transformation is approximately 0.5 m, although in some areas it is in the order of 2 m.

6.2 The 2D transformation

The 2D transformation method for EUREF-FIN and kkj coordinates is described in JHS154. Only the basic concept is, however, described; the actual transformation parameters are not given.

The 2D transformation is based on the affine transformation. The basic formulas are as follows:

$$x_2 = \Delta x + a_1x_1 + a_2y_1$$
$$y_2 = \Delta y + b_1x_1 + b_2y_1$$

The Finnish territory and areas close to the international borders of Finland are divided into triangles (by means of Delaunay triangulation). The corner points of each triangle have known coordinates in both systems assigned to them, i.e. in kkj (zone 3, central meridian 27°E) and EUREF-FIN (or more precisely ETRS-GK27). There are 624 actual points and 143 virtual points (outside the Finnish borders), which are used to determine the transformation. With those known points, the translations (Δx,Δy), the rotation angle and the scale differences are determined for each triangle. Factors a_1, a_2, b_1 and b_2 are functions of the rotation angle and the scale differences.

When the coordinates of one point are to be transformed, the triangle in which the point is located must first be determined. The above transformation formula is then used with the parameters Δx, Δy, a_1, a_2, b_1 and b_2 determined for that particular triangle. The accuracy of a transformed coordinate is on average better than 10 cm.

The transformation program, a list of the triangles and the coordinates of the points can be found on the website of National Land Survey http://www.maanmittauslaitos.fi/Default.asp?id=102 (in Finnish).

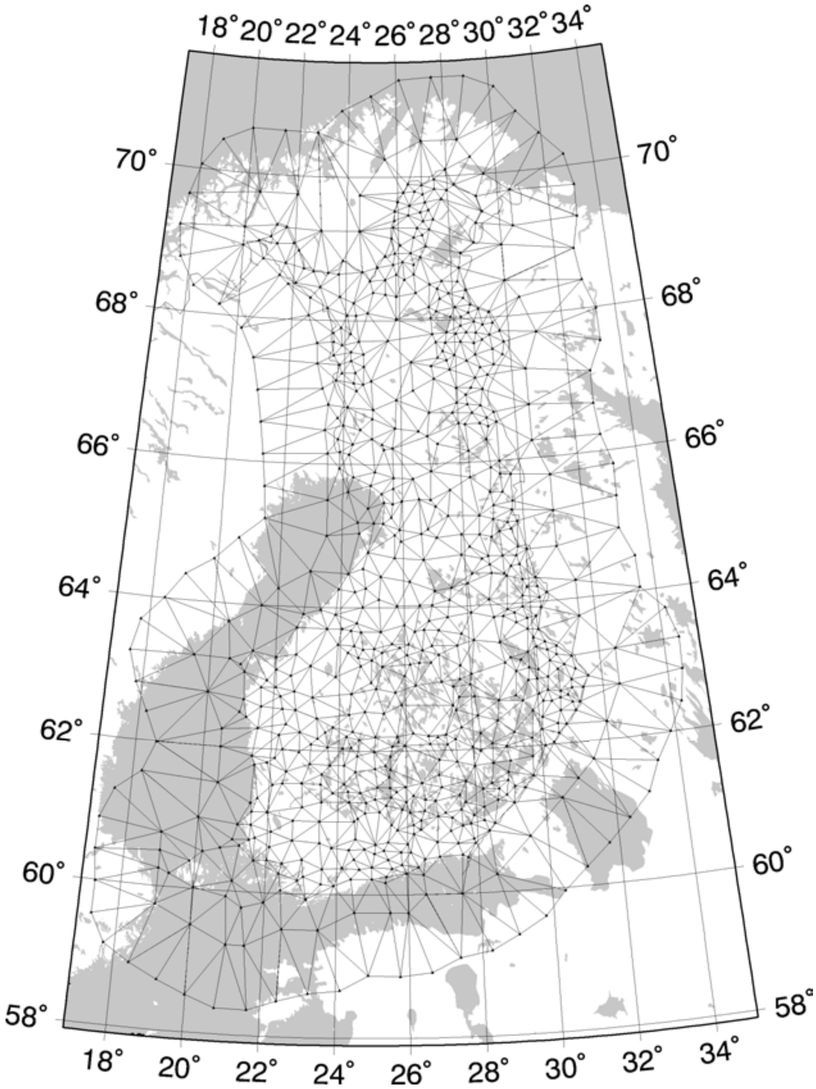

Figure 2. The points and triangles used in the transformation

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